

AD-A014 781

**EFFECT OF SLEWING ON THE FREQUENCY SPECTRUM OF A  
LASER BEAM IN A TURBULENT MEDIUM**

**Ronald L. Fante**

**Air Force Cambridge Research Laboratories  
Hanscom Air Force Base, Massachusetts**

**8 May 1975**

**DISTRIBUTED BY:**

**NTIS**

**National Technical Information Service  
U. S. DEPARTMENT OF COMMERCE**

268192

AFCRL-TR-75-0259  
PHYSICAL SCIENCES RESEARCH PAPERS, NO. 630



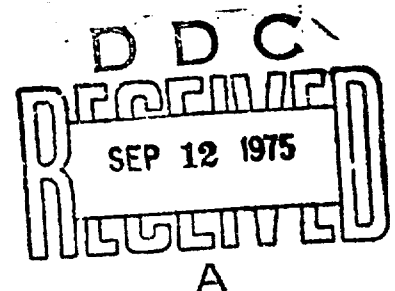
AD A014781

## Effect of Slewing on the Frequency Spectrum of a Laser Beam in a Turbulent Medium

RONALD L. FANTE

8 May 1975

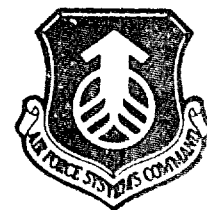
Approved for public release; distribution unlimited.



MICROWAVE PHYSICS LABORATORY    PROJECT 2153  
**AIR FORCE CAMBRIDGE RESEARCH LABORATORIES**  
HANSCOM AFB, MASSACHUSETTS 01731

**AIR FORCE SYSTEMS COMMAND, USAF**

Reproduced by  
NATIONAL TECHNICAL  
INFORMATION SERVICE  
U.S. Department of Commerce  
Springfield, VA. 22151



Qualified requestors may obtain additional copies from the Defense Documentation Center. All others should apply to the National Technical Information Service.

ACQUISITION	
DTIC	Write Section
NSC	Buy Section
UNCLASSIFIED	
CLASSIFICATION	
DISTRIBUTION/AVAILABILITY CODES	
AVAIL	AVAIL. 240/4 SPECIAL
A	

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFCRL-TR-75-0259	2. GOVT ACCESSION NO.	3. REPORTING CATALOG NUMBER
4. TITLE (and Subtitle) EFFECT OF SLEWING ON THE FREQUENCY SPECTRUM OF A LASER BEAM IN A TURBULENT MEDIUM		5. TYPE OF REPORT & PERIOD COVERED Scientific. Interim.
7. AUTHOR(s) Ronald L. Fante		6. PERFORMING ORG. REPORT NUMBER PSRP No. 630
9. PERFORMING ORGANIZATION NAME AND ADDRESS Air Force Cambridge Research Laboratories (LZP) Hanscom AFB Massachusetts 01731		8. CONTRACT OR GRANT NUMBER(s) 21530201, 61102F, 681305
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Cambridge Research Laboratories (LZP) Hanscom AFB Massachusetts 01731		10. PROGRAM ELEMENT, PROJECT, TASK AND MONITORING UNIT NUMBERS
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		12. REPORT DATE 8 May 1975
		13. NUMBER OF PAGES 10
		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES TECH, OTHER		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Propagation Atmospheric optics Lasers Turbulence		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) In this report, we have studied the effect of beam slewing on the amplitude and phase spectra of a laser beam propagating in a turbulent medium.		

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

## Contents

1. INTRODUCTION	5
2. SLEWING LASER BEAM	5

## Illustrations

1. Geometry of a Slewng Laser Beam	6
2. Temporal Frequency Spectrum of the Amplitude Fluctuations at the Center of a Laser Beam Propagating in Turbulence for the Case When $F = 10^{-7}$ and $a = 1$	10
3. Temporal Frequency Spectrum of the Phase Fluctuations at the Center of a Laser Beam Propagating in a Turbulent Medium for the Case When $F = 10^{-7}$ and $a = 1$	10

## Effect of Slewing on the Frequency Spectrum of a Laser Beam in a Turbulent Medium

### 1. INTRODUCTION

In this report, we study the effect of slewing on the temporal frequency spectrum of the scintillations of a laser beam propagating through a turbulent medium. We shall assume that the turbulent medium is characterized by the index of refraction

$$n(\underline{r}, t) = 1 + n_1(\underline{r}, t) , \quad (1)$$

where the fluctuation  $n_1 \ll 1$ . It is further assumed that  $n_1$  varies sufficiently slowly in comparison with the time required for the signal traversal.

### 2. SLEWING LASER BEAM ANALYSIS

Let us consider a laser beam propagating in the  $x$  direction in a turbulent medium which, in the absence of any slewing of the beam, has a transverse wind velocity given by  $\underline{V}(x)$ . If we assume that the flow of the turbulence is frozen, so that the medium doesn't change within the time of the signal propagation through it—except for the shift due to the wind—it is easily seen that

(Received for publication 7 May 1975)

$$n_1(\underline{r}, t) = (\underline{r} - \underline{V}t, 0) . \quad (2)$$

Now let us suppose that the beam is slewed at a constant angular rate  $\omega_s$ . Then, in a coordinate system moving with the slewing beam, it appears that the turbulent eddies are moving relative to the beam with a transverse velocity

$$\underline{v}_e = V(x) \cos \theta + \omega_s \underline{x} , \quad (3)$$

where  $\theta$  is defined in Figure 1 and  $\omega_s$  is positive when the beam is slewing in a direction opposite to that of the ambient wind. In this case, for a coordinate system moving with the slewing beam we have

$$n_1(\underline{r}, t) = n_1(\underline{r} - \underline{v}_e t, 0) . \quad (4)$$

The electric field in a coordinate system moving with the slewing beam can be written quite generally as

$$E(\underline{r}, t) = A e^{X(\underline{r}, t) + iS(\underline{r}, t)} , \quad (5)$$

where  $X$  is the logarithm of the amplitude,  $S$  is the phase fluctuation, and  $A$  is a determinate quantity, which is independent of the turbulent fluctuations. The

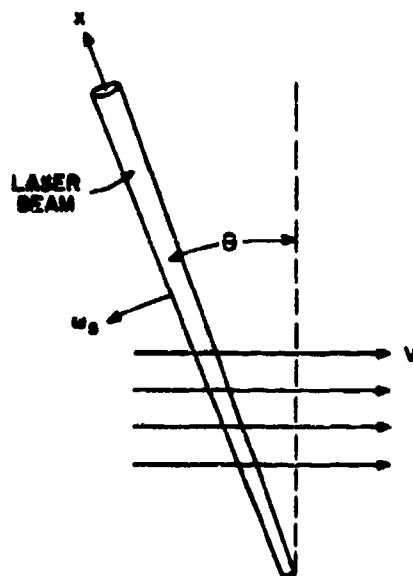


Figure 1. Geometry of a Slewing Laser Beam

correlation functions  $B_x(\underline{r}_1, \underline{r}_2, \tau)$ ,  $B_s(\underline{r}_1, \underline{r}_2, \tau)$  of the log amplitude and phase fluctuations are defined as

$$B_x(\underline{r}_1, \underline{r}_2, \tau) = \langle x(\underline{r}_1, t) x(\underline{r}_2, t+\tau) \rangle \quad (6)$$

and

$$B_s(\underline{r}_1, \underline{r}_2, \tau) = \langle S(\underline{r}_1, t) S(\underline{r}_2, t+\tau) \rangle \quad (7)$$

where  $\langle \rangle$  denotes an ensemble average. We assume that the transmitter field at  $x = 0$  is given by (collimated beam)

$$E(x=0, \underline{\rho}) = e^{-\rho^2/W_0^2} \quad (8)$$

where  $\underline{\rho} = (y, z)$ . It can then be shown that for a point located at the center of the beam at a distance  $x = L$  from the transmitter, the correlation functions  $B_x$  and  $B_s$  are given by

$$B_{\chi_s}(\tau) = \pi^2 \int_0^L d\eta \int_0^\infty \kappa d\kappa J_0(\kappa |\underline{\rho}_s \tau|) \left[ 2 |H|^2 \pm H^2 \pm H^{*2} \right] \Phi(\kappa) \quad (9)$$

where the upper signs are for  $B_x$  and the lower for  $B_s$ . Also

$$|H|^2 = k^2 \exp \left\{ -\frac{\gamma_2(L-\eta)\kappa^2}{k} \right\},$$

$$H^2 = -k^2 \exp \left\{ -i(\gamma_1 - i\gamma_2) \frac{(L-\eta)\kappa^2}{k} \right\},$$

$$\gamma_1 = \frac{1 + \alpha_1^2 L \eta}{1 + \alpha_1^2 L^2},$$

$$\gamma_2 = \frac{\alpha_1(L-\eta)}{1 + \alpha_1^2 L^2},$$

$$\alpha_1 = \frac{\lambda}{\pi W_0^2},$$

1. Ishimaru, A. (1969) Fluctuations of a focused beam wave for atmospheric turbulence probing, Proc. IEEE 57:407-414.

$\lambda$  = signal wavelength,

and

$$k = 2\pi/\lambda.$$

The function  $\Phi(k)$  is the wavenumber spectrum of the index-of-refraction fluctuations and is given by

$$\Phi(k) = \frac{0.033 C_n^2 \exp(-k^2 l_0^2)}{[k^2 + L_0^{-2}]^{11/6}}, \quad (10)$$

where  $L_0$  is the outer scale size of the turbulent eddies,  $l_0 = 2\pi r_0$  is the inner scale size of the eddies, and  $C_n^2$  is the index of refraction structure constant. Equation (9) is valid only when the turbulence is weak. In particular, we require that

$$\sigma_1^2 = 1.23 k^{7/6} C_n^2 L^{11/6} \ll 1.$$

We further require that  $L \ll c/\omega_s$  where  $c$  is the speed of light.

By observing the result in Eq. (9) it is evident that for frozen-in turbulence, the variance of the log-amplitude and phase fluctuations is independent of both the windspeed and the slewing rate. That is  $\langle \chi^2 \rangle = B_\chi(0)$  and  $\langle S^2 \rangle = B_S(0)$  are independent of  $\underline{v}_s$  as is seen by setting  $\tau = 0$  in Eq. (9). The temporal frequency spectrum of the fluctuations, however, does depend on both the wind velocity and the slewing rate. The temporal frequency spectra  $W_\chi(\omega)$  and  $W_S(\omega)$  are given by

$$\begin{pmatrix} W_\chi(\omega) \\ W_S(\omega) \end{pmatrix} = \int_0^\infty d\tau \cos \omega \tau \begin{pmatrix} B_\chi(\tau) \\ B_S(\tau) \end{pmatrix}. \quad (11)$$

If Eq. (9) is used in (11) we get, after performing the integration on  $\tau$ , and assuming  $\underline{V}(\underline{x})$  is independent of position and that the turbulence is homogeneous

$$\left\{ \begin{matrix} W_x(\Omega) \\ W_s(\Omega) \end{matrix} \right\} = A_0 \int_0^1 d\xi \int_{\left(\frac{\Omega}{1+\gamma\xi}\right)^2}^{\infty} \frac{ds \exp \left[ -s(1-\xi)^2 \left( \frac{1+a^2\xi}{1+a^2} \right) \right]}{[s+F]^{11/6} [(1+\gamma\xi)^2 s + \Omega^2]^{1/2}} \times \begin{cases} \sin^2 \left[ \frac{s(1-\xi)}{2} \left( \frac{1+a^2\xi}{1+a^2} \right) \right] \\ \cos^2 \left[ \frac{s(1-\xi)}{2} \left( \frac{1+a^2\xi}{1+a^2} \right) \right] \end{cases} \quad (12)$$

where

$$v_0 = V \cos \theta,$$

$$F = \frac{L}{kL_0^2},$$

$$a = \frac{\lambda L}{\pi W_0^2},$$

$$\gamma = \frac{\omega}{v_0} \left( \frac{L}{k} \right)^{1/2},$$

$$A_0 = 0.528 \sigma_1^2 \left( \frac{L}{kv_0^2} \right)^{1/2},$$

and

$$\gamma = \frac{\omega_s L}{v_0}$$

Equation (12) has been evaluated numerically for the frequency spectra  $W_x(\Omega)$  and  $W_s(\Omega)$  for a number of different values of the slewing parameter  $\gamma$ ; the results are presented in Figures 2 and 3. We note, as expected, that the spectral width of the amplitude and phase fluctuations increase as the slewing rate in the direction opposite to the turbulent flow is increased. Therefore, even though slewing does not affect the values of

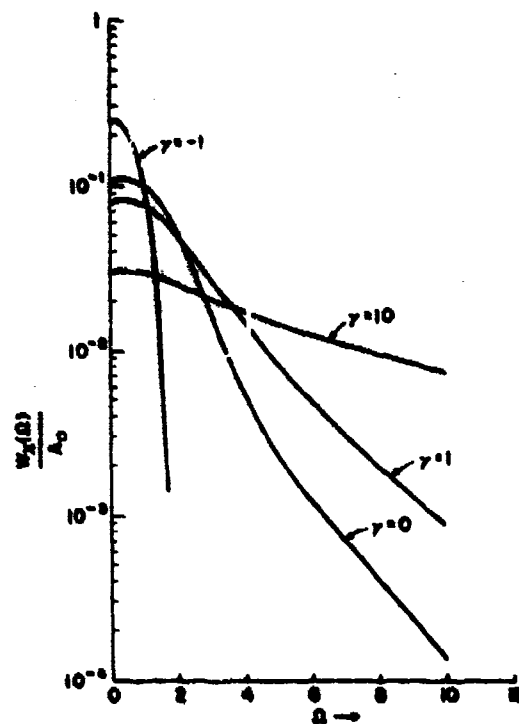


Figure 2. Temporal Frequency Spectrum of the Amplitude Fluctuations at the Center of a Laser Beam Propagating in Turbulence for the Case When  $F = 10^{-7}$  and  $a = 1$ . Note that  $\gamma$  is positive when the beam is slewed in the direction opposite to  $V$

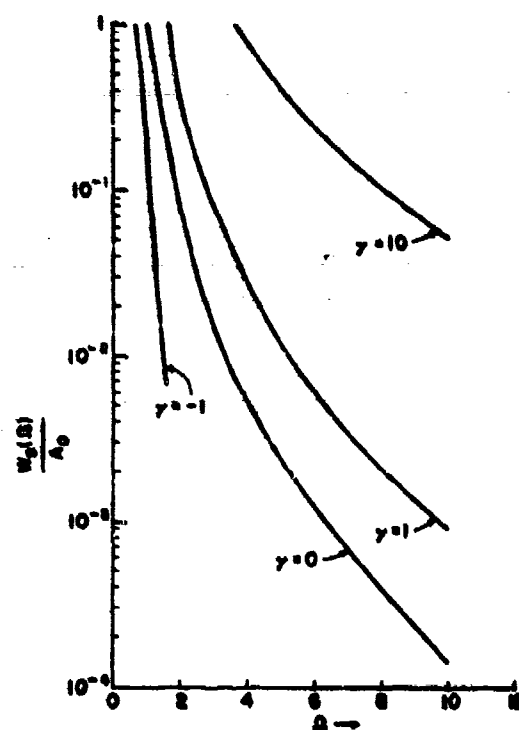


Figure 3. Temporal Frequency Spectrum of the Phase Fluctuations at the Center of a Laser Beam Propagating in a Turbulent Medium for the Case When  $F = 10^{-7}$  and  $a = 1$ . Note that  $\gamma$  is positive when the beam is slewed in the direction opposite to  $V$

$$\langle \chi^2 \rangle = \frac{1}{\pi} \int_{-\infty}^{\infty} W_{\chi}(\omega) d\omega$$

and

$$\langle \phi^2 \rangle = \frac{1}{\pi} \int_{-\infty}^{\infty} W_{\phi}(\omega) d\omega .$$

it does affect the shape of  $W_{\chi}$  and  $W_{\phi}$ .